Practice Problem for Limited Dependent Variables

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The following is based on a question from the 2020 "PhD Application Exam."

1. Let $(y_1, x_1), \cdots, (y_N, x_N)$ be a collection of iid observations where $y_i \in \{0, 1\}$ and x_i is continuously distributed. Suppose that $p(x_i) \equiv \mathbb{P}(y_i = 1|x_i) = F(\alpha + \beta x_i)$ where $F(z) = e^z/(1 + e^z)$ and (α, β) are unknown parameters.
   (a) Derive an expression for the partial effect of x_i on $p(x_i)$ in this model.
   (b) Write out the log-likelihood function $\ell_N(\alpha, \beta)$ for this model, simplifying your result as far as possible.
   (c) Using your answer to the preceding part, derive the first-order conditions for the maximum likelihood estimators of α and β. Simplify your results as far as possible.