Identification!

Lewbel (2019) "Identification Zoo"
⇒ JEL ⇐

sample! $\hat{\beta} = \left(\frac{\underset{\sim}{X}'\underset{\sim}{X}}{n}\right)^{-1}\left(\frac{\underset{\sim}{X}'\underline{y}}{n}\right) = \left(\frac{1}{n}\sum_{i=1}^{n}\underline{x}_i\underline{x}_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n}\underline{x}_i y_i\right)$

popn $\boxed{\underline{\beta} = \mathbb{E}(\underline{x}_i\underline{x}_i')^{-1}\,\mathbb{E}(\underline{x}_i y)}$

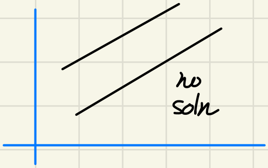
$\boxed{\mathbb{E}(\underline{x}_i\underline{x}_i')\,\underline{\beta} = \mathbb{E}(\underline{x}_i y)}$

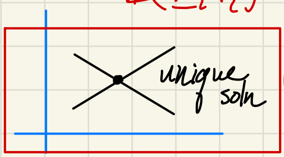
$\underset{\sim}{A}\,\underline{\beta} = \underline{v}$

Moment Conditions: $\mathbb{E}(\underline{x}_i\epsilon_i) = 0$

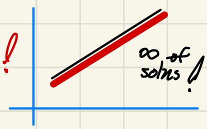
$\mathbb{E}\left[\underline{x}_i(Y_i - \underline{x}_i'\underline{\beta})\right] = 0$

$\mathbb{E}(\underline{x}_i Y_i) - \mathbb{E}(\underline{x}_i\underline{x}_i')\underline{\beta} = 0$





β is (point) identified!



$\underset{\sim}{X} = \begin{pmatrix}\underline{x}_1' \\ \vdots \\ \underline{x}_n'\end{pmatrix}$

$\underset{\sim}{X}'\underset{\sim}{X} = (\underline{x}_1 \ldots \underline{x}_n)\begin{pmatrix}\underline{x}_1' \\ \vdots \\ \underline{x}_n'\end{pmatrix}$

$\sum_{i=1}^{n}\underline{x}_i\underline{x}_i'$

$\underset{\sim}{X}'\underline{y} = (\underline{x}_1 \ldots \underline{x}_n)\begin{pmatrix}y_1 \\ \vdots \\ y_n\end{pmatrix} = \sum_{i=1}^{n}\underline{x}_i y_i$

Index Model : $\mathbb{P}(Y=1|\underline{x}) = \mathbb{E}(Y|\underline{X}) = G(\underline{x}'\underline{\beta})$

KNOWN function

parametric model!

When is $\underline{\beta}$ point identified? When can we solve uniquely for $\underline{\beta}$ using the whole dist of (Y, X)

G function has some properties:

1. G strictly increasing $\Longrightarrow$ G^{-1}!
2. bounded between 0 and 1

$$\mathbb{E}(Y|\underline{X}_i) = G(\underline{X}_i'\underline{\beta})$$

$$G^{-1}(\mathbb{E}(Y_i|\underline{X}_i)) = \cancel{G}^{-1}(\cancel{G}(\underline{X}_i'\underline{\beta}))$$

$$G^{-1}(\mathbb{E}(Y_i|\underline{X}_i)) = \underline{X}_i'\underline{\beta}$$

$$\underline{X}_i G^{-1}(\mathbb{E}(Y_i|\underline{X}_i)) = \underline{X}_i\underline{X}_i'\underline{\beta}$$

$$\mathbb{E}\{\underline{X}_i G^{-1}(\mathbb{E}(Y_i|\underline{X}_i))\} = \boxed{\mathbb{E}(\underline{X}_i\underline{X}_i')}\underline{\beta}$$

assume invertible

$$\boxed{\underline{\beta} = \mathbb{E}(\underline{X}_i\underline{X}_i')^{-1}\mathbb{E}\{\underline{X}_i G^{-1}(\boxed{\mathbb{E}(Y_i|\underline{X}_i)})\}}$$

observed!

suppose you have the WHOLE POPN!
$\Longrightarrow$ any feature of dist (Y, X) is known!

Joint indep

$X_1, \dots, X_J$ are JOINTLY indep.

iff $\quad \mathbb{P}(X_1, \dots, X_J) = \mathbb{P}(X_1)\mathbb{P}(X_2)\cdots\mathbb{P}(X_J)$

Implication: X_j indep of X_k for any $j \neq k$

need $\mathbb{P}(X_1, X_2) = \mathbb{P}(X_1)\,\mathbb{P}(X_2)$

$$f(x_1, x_2) = \int\cdots\int \underbrace{f(x_1, x_2, \dots, x_J)}_{\left(f(x_1)\cdots f(x_J)\right)}\, dx_3\, dx_4 \cdots dx_J$$

$$f(x_1)f(x_2)\underbrace{\int\cdots\int \underbrace{\boxed{f(x_3)\cdots f(x_J)}}_{f(x_3, \dots, x_J)}\, dx_3 \cdots dx_J}_{1}$$

Questions after class

⇒ 2 typos (fixed)

⇒ $(V_1, V_2) \perp\!\!\!\perp X \Rightarrow \begin{array}{l} V_1 \perp\!\!\!\perp X \\ V_2 \perp\!\!\!\perp X \end{array}$

⇒ Suppose $\lambda(\cdot)$ were a linear function but we had an exclusion restriction. Would the coef. on x_1 in the Heckman selection model be identified? YES: δ coefs are KNOWN from estimating participation eq.

$\boxed{\text{The question I kind of flubbed during lecture...}}$

Why doesn't this work?

$$\boxed{\begin{aligned} \underline{X}_i \underline{X}_i' \underline{\beta} &= \underline{X}_i Y_i \\ \underline{\beta} &= (\underline{X}_i \underline{X}_i')^{-1} \underline{X}_i Y_i \\ \underline{\beta} &= \mathbb{E}\left[(\underline{X}_i \underline{X}_i')^{-1} \underline{X}_i' Y_i\right] \end{aligned}}$$

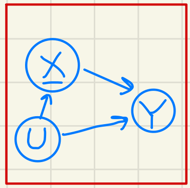
This equation is WRONG

How can we tell? $Y_i = \underline{X}_i' \underline{\beta} + U_i \; ; \begin{cases} \mathbb{E}(U_i) = 0 \\ \mathbb{E}(\underline{X}_i U_i) = 0 \end{cases}$

$$\Longrightarrow \quad \underline{X}_i Y_i = \underline{X}_i \underline{X}_i' \underline{\beta} + \underline{X}_i U_i$$

This DOESN'T EQUAL ZERO! (It's a RV w/ mean equal to zero.)

$$\cancel{Y_i = \underline{X}_i'\beta + U_i}$$



?

causal model?

$$\boxed{U_i \equiv \boxed{Y_i} - \boxed{\underline{X}_i'}\beta \quad \text{where} \quad \underbrace{\beta \equiv \mathbb{E}(\underline{X}_i\underline{X}_i')^{-1}\,\mathbb{E}(\underline{X}_iY_i)}}$$

RVs in a popn.

popn linear regression model

$$\mathbb{E}\left[\underline{X}_i(Y_i - \underline{X}_i'\beta)\right] = \underline{0}$$

$\Longrightarrow$ Suppose $\underline{X}_i = \begin{pmatrix} 1 \\ X_{i1} \\ \vdots \\ X_{iK} \end{pmatrix}$

$$\mathbb{E}\left[\begin{pmatrix} 1 \\ x_{i1} \\ \vdots \\ x_{iK} \end{pmatrix}\underbrace{\left\{Y_i - \begin{pmatrix} 1 \\ x_{i1} \\ \vdots \\ x_{iK} \end{pmatrix}'\begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_K \end{pmatrix}\right\}}_{U_i}\right]$$

$$\mathbb{E}(U_i) = 0, \quad \mathbb{E}(\underline{X}_i U_i) = \underline{0}$$

$$\boxed{\underline{X}_i\underline{X}_i'\beta \overset{?}{=} \underline{X}_iY_i}$$

NO

$$\boxed{\underline{X}_i'\beta} = Y_i \quad \text{mult both sides by } \underline{X}_i'$$

$$\mathbb{E}\left[(\underline{X}_i\underline{X}_i')^{-1}\,\underline{X}_iY_i\right] \overset{?}{=} \mathbb{E}(\underline{X}_i\underline{X}_i')^{-1}\,\mathbb{E}(\underline{X}_iY_i)$$

$$\mathbb{E}\left(\frac{1}{X^2}\,XY\right) \overset{?}{=} \mathbb{E}(X^2)^{-1}\,\mathbb{E}(XY)$$

$$\mathbb{E}(Y/X) = \frac{1}{\mathbb{E}(X^2)}\,\mathbb{E}(XY)$$

Heckman Selection

Make sure you understand the BIG Picture!

what is the problem w/ ignoring selection?

Lemma 1: $\mathbb{E}(y_1|\mathbf{x}, y_2 = 1) = \mathbf{x}_1'\beta_1 + \boxed{\gamma_1 \mathbb{E}(v_2|\mathbf{x}, y_2 = 1)}$ NOT zero and depends on $\underline{x}$!

- Shorthand: $\boxed{h(\mathbf{x}) \equiv \mathbb{E}(v_2|\mathbf{x}, y_2 = 1)}$ *
- (β_1, γ_1) identified from regression of y_1 on $[\mathbf{x}_1, h(\mathbf{x})]$ for selected population.

Outcome Equation

$y_1 = \mathbf{x}_1'\beta_1 + u_1$

Participation Equation

$y_2 = \mathbb{1}\{\mathbf{x}'\delta_2 + v_2 > 0\}$

Assumptions

(a) Observe $y_2, \mathbf{x}' = (\mathbf{x}_1', \mathbf{x}_2')$; only observe y_1 if $y_2 = 1$.

(b) (u_1, v_2) are mean zero and jointly independent of $\mathbf{x}$.

(c) $v_2 \sim \text{Normal}(0, 1)$

(d) $\mathbb{E}(u_1|v_2) = \gamma_1 v_2$ where γ_1 is an unknown constant.

Notes

$\boxed{\text{IPW}}$ ← another way to address selection

post-stratification

Horvitz-Thompson